OCR
Oxford Cambridge and RSA

GCE

Mathematics A

H240/02: Pure Mathematics and Statistics

A Level

Mark Scheme for June 2023

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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MARKING INSTRUCTIONS

PREPARATION FOR MARKING

RM ASSESSOR

1. Make sure that you have accessed and completed the relevant training packages for on-screen marking: *RM Assessor Online Training*; *OCR Essential Guide to Marking*.

2. Make sure that you have read and understood the mark scheme and the question paper for this unit. These are posted on the RM Cambridge Assessment Support Portal http://www.rm.com/support/ca

3. Log-in to RM Assessor and mark the **required number** of practice responses ("scripts") and the **number of required** standardisation responses.

MARKING

1. Mark strictly to the mark scheme.

2. Marks awarded must relate directly to the marking criteria.

3. The schedule of dates is very important. It is essential that you meet the RM Assessor 50% and 100% (traditional 40% Batch 1 and 100% Batch 2) deadlines. If you experience problems, you must contact your Team Leader (Supervisor) without delay.

4. **Annotations**

Annotation	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank Page
Seen	
Highlighting	

Other abbreviations in mark scheme	Meaning
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only one previous M mark
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

5. **Subject Specific Marking Instructions**

a. Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ^) is sufficient, but not required.

For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

Award NR (No Response)
- if there is nothing written at all in the answer space and no attempt elsewhere in the script
- OR if there is a comment which does not in any way relate to the question (e.g. 'can't do', 'don't know')
- OR if there is a mark (e.g. a dash, a question mark, a picture) which isn't an attempt at the question.

Note: Award 0 marks only for an attempt that earns no credit (including copying out the question).

If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of doubt unless it is ambiguous for which part it is intended.

b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

If you are in any doubt whatsoever you should contact your Team Leader.

c. The following types of marks are available.

M
A suitable method has been selected and applied in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.
A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words "Determine" or "Show that", or some other indication that the method must be given explicitly.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d. When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e. The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f. We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.
   - When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value.
   - When a value is not given in the paper accept any answer that agrees with the correct value to 3 s.f. unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range.
     NB for Specification B (MEI) the rubric is not specific about the level of accuracy required, so this statement reads "2 s.f".

   Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.
   Candidates using a value of 9.80, 9.81 or 10 for g should usually be penalised for any final accuracy marks which do not agree to the value found with 9.8 which is given in the rubric.

g. Rules for replaced work and multiple attempts:
   - If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.
   - If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.
   - if a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.

h. For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as cao may be awarded as long as there are no other errors.
   If a candidate corrects the misread in a later part, do not continue to follow through. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i. If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers, provided that there is nothing in the wording of the question specifying that analytical methods are required such as the bold "In this question you must show detailed reasoning", or the command words "Show" or "Determine". Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j. If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Mark	AO	Guidance
1	(a)	(i)	$(x-4)^2-5$	B1 B1 [2]	1.1 1.1	B1 for each constant. For '4' (Allow $a=4$) For '−5' (Allow $b=-5$) ISW
1	(a)	(ii)	-5	B1FT [1]	1.1	FT their b Accept (4, −5) as the coordinates of the minimum point but must be correct for their a and b.
1	(b)		$(-8)^2-4\times(11-k)=0$ or $x^2-8x+11-k \equiv (x-4)^2$ $k=-5$	M1 A1 [2]	1.1 1.1	Write $b^2-4ac=0$ (accept >0 for this mark only) or equate with completed square form. Accept a sketch or equivalent reasoning. Final answer must be given as $k=-5$ SC**B1**: correct answer without working (max [1/2])

Question			Answer	Mark	AO	Guidance
2	(a)		$10\|k\|$	B1 [1]	1.1	Allow $10k$
2	(b)		$\overrightarrow{BP} = \begin{pmatrix} 6k-1 \\ -2 \\ 8k-3 \end{pmatrix}$ or $\overrightarrow{PB} = \begin{pmatrix} 1-6k \\ 2 \\ 3-8k \end{pmatrix}$	B1	3.1a	May be implied (but must be fully correct to imply, i.e. including (-2))
			$100k^2 + (6k-1)^2 + (-2)^2 + (8k-3)^2 = 14$	M1	1.1	Attempt $OP^2 + BP^2 = OB^2$ FT their OP and $\overrightarrow{BP}$ or $\overrightarrow{PB}$
			$200k^2 - 60k + 14 = 14$	A1	1.1	Correct equation after expanding brackets
			$k = \frac{3}{10}$	A1	1.1	oe Condone inclusion of $k = 0$ (whether eliminated or not)
			Alternative method using scalar product $\overrightarrow{BP} = \begin{pmatrix} 6k-1 \\ -2 \\ 8k-3 \end{pmatrix}$ or $\overrightarrow{PB} = \begin{pmatrix} 1-6k \\ 2 \\ 3-8k \end{pmatrix}$	B1		May be implied
			$6k(6k-1) + 0 + 8k(8k-3) = 0$	M1		Attempt $\overrightarrow{OP} . \overrightarrow{BP} = 0$ FT their $\overrightarrow{OP}$ and $\overrightarrow{BP}$ or $\overrightarrow{PB}$ (or attempt $\overrightarrow{OA} \cdot \overrightarrow{BP} = 0 \Rightarrow 6(6k-1) + 0 + 8(8k-3) = 0$) Must be algebraic – i.e. an equation of this form in k.
			$100k^2 - 30k = 0$	A1		Correct equation after expanding brackets Or $100k - 30 = 0$
			$k = \frac{3}{10}$	A1		oe Condone inclusion of $k = 0$ (whether eliminated or not)
				[4]		

Question			Answer	Mark	AO	Guidance
3						**DR**
			$\int \frac{1}{x+2}\,dx = \ln(x+2)$	**M1**	**1.1**	Integrate and obtain expression involving ln. This may be combined with the next step. Ignore limits. Brackets (x+2) soi If a substitution is used then only award this mark when $\ln(x+2)$ reached or an equivalent integral with appropriate limits (typically $\ln x$ with limits 2 and 4.5).
			$\left[\ln(x+2)\right]_0^{2.5} = \ln 4.5 - \ln 2$ or $\ln\frac{4.5}{2}$	**M1**	**1.1**	Substitute and use limits 0 & 2.5 (or appropriately changed limits in the case of a substitution) in their ln integral. Must see this step.
			$= \ln\frac{9}{4}$ or $\ln 2.25$ or $2\ln\left(\frac{3}{2}\right)$ etc	**A1**	**1.1**	www, any equivalent exact form, but must be a single term expression. Do not accept $\ln\frac{4.5}{2}$ for this mark.
				[3]		Correct answer with no working SC**B1** [1/3] Ignore use of modulus signs throughout.

Question			Answer	Mark	AO	Guidance
4	(a)		$\frac{dy}{dx} = 2x$.	M1	3.1a	
			Gradient of tangent at (1, 1) is 2 Gradient of normal at (1, 1) $= -\frac{1}{2}$	A1	1.1	For correct gradient of either tangent or normal
			Equation of normal is $y - 1 = -\frac{1}{2}(x - 1)$	M1*	1.1	Using their gradient of normal and (1,1) to form an equation or $y = -\frac{1}{2}x + \frac{3}{2}$ (must find a value for c for this mark to be awarded) FT their gradient of normal (not = 2)
			At B: $x^2 = -\frac{1}{2}x + \frac{3}{2}$	M1 dep*	3.1a	Substituting $y = x^2$ and attempting to solve (may see quadratic in x or y)
			$2x^2 + x - 3 = 0$	A1	1.1	Correct quadratic in x or y, in any form or $4y^2 - 13y + 9 = 0$
			B is $(-\frac{3}{2}, \frac{9}{4})$	A1	1.1	Allow (-1.5,2.25). Accept x=…, y=… but must see both.
				[6]		
4	(b)		$AB^2 = \left(1 - \left(-\frac{3}{2}\right)\right)^2 + \left(1 - \frac{9}{4}\right)^2 \quad \left(= \frac{125}{16}\right)$ $OB^2 = \left(\frac{3}{2}\right)^2 + \left(\frac{9}{4}\right)^2 \quad \left(= \frac{117}{16}\right)$ $OA^2 = 2$			or $AB = \frac{5\sqrt{5}}{4}$ or $OB = \frac{3\sqrt{13}}{4}$ or $OA = \sqrt{2}$
				M1	2.1	Attempt to find all three, squared or not (may see on diagram)
			$\cos\alpha = \frac{\frac{117}{16} + 2 - \frac{125}{16}}{2 \times \sqrt{\frac{117}{16}} \times \sqrt{2}}$	M1 dep	1.1	Correct use of cos rule in any form, FT their AB, OB & OA
			$\cos\alpha = \frac{1}{\sqrt{26}}$ OR $\tan\alpha = \frac{\sqrt{26-1}}{1}$ $\tan\alpha = 5$	A1	1.1	Must be exact and come from exact working for cos α. Must see an exact intermediate step – either $\frac{1}{\sqrt{26}}$ or $\frac{\sqrt{26-1}}{1}$. Decimals used throughout can achieve max [2/3] M1M1A0 Do not penalise candidates who write down α =78.69…° but the final answer must come from exact working (not BC).

			Alternative method (1) $tan\,\beta = \frac{9}{4} \div \frac{3}{2} = \frac{3}{2}, \tan\gamma = 1$ $\tan\alpha = -\tan(\beta+\gamma) = -\frac{\frac{3}{2}+1}{1-\frac{3}{2}\times 1}$ $= 5$	**M1** **M1 dep** **A1**		Attempt to find $\tan\beta$ & $\tan\gamma$ β and γ are the angles made by OB and OA respectively with the horizontal (Alternatively with angles to the vertical, $\tan\beta = \frac{2}{3}$) Correct use of $\tan(180-\theta) = -\tan\theta$ and $\tan(A+B)$ formula using their $\tan\beta$ & $\tan\gamma$ (or, with angles to the vertical, $\tan\alpha = \frac{1+\frac{2}{3}}{1-\frac{2}{3}\times 1}$) Must be exact and come from exact working. Do not penalise candidates who write down α =78.69…° but the final answer must come from exact working (not BC).
			Alternative method (2) $\tan BOx = \frac{9}{4} \div (-\frac{3}{2}) = -\frac{3}{2}$, $\tan AOx = 1$ $\tan(BOx - AOx)$ $= \frac{-\frac{3}{2}-1}{1+(-\frac{3}{2})\times 1}$ $= 5$	 **M1** **M1 dep** **A1**		 Attempt find $\tan BOx$ & $\tan AOx$ and use $\tan(\theta-\varphi)$ formula Correct use of $\tan(\theta-\varphi)$ using their $\tan BOx$ & $\tan AOx$ Must be exact and come from exact working.
			Alternative method (3) $\overrightarrow{OB}\cdot\overrightarrow{OA} = -\frac{3}{2}+\frac{9}{4} = \sqrt{2}\times\frac{3\sqrt{13}}{4}\cos\alpha$ $\cos\alpha = \frac{\frac{3}{4}}{\sqrt{2\times\frac{117}{16}}}$ $\cos\alpha = \frac{1}{\sqrt{26}}$ OR $\tan\alpha = \frac{\sqrt{26-1}}{1}$ $\tan\alpha = 5$	**M1** **M1 dep** **A1**		Attempt $\overrightarrow{OB}\cdot\overrightarrow{OA}$ Correct dot product with exact values Must be exact and come from exact working for cos α. Must see an exact intermediate step – either $\frac{1}{\sqrt{26}}$ or $\frac{\sqrt{26-1}}{1}$.
				[3]		

Question			Answer	Mark	AO	Guidance
5	(a)	(i)	$f'(x) = -\sin x + \sqrt{3}\cos x$	M1	3.1a	Attempt differentiate f(x), allow sign errors but both trig functions must be changed. oe e.g. $2\cos\left(x + \frac{\pi}{6}\right)$
			$\tan x = \sqrt{3}$	M1	1.1	Setting their $f'(x) = 0$ and correctly manipulating to reach an equation in a single trig function (e.g. $4\cos^2 x = 1$ or $4\sin^2 x = 3$) do not allow incorrect working e.g. $a - b = 0 \Rightarrow a^2 - b^2 = 0$
			$x = \frac{1}{3}\pi$	A1	1.1	www, must be in radians (allow decimal 1.05 3sf)
			or $\frac{4}{3}\pi$	A1	1.1	www, must be in radians (allow decimal 4.19 3sf), and for this mark have no other solutions in the given range (ignore any outside) SC**B1** for each correct solution (max [2/4]) if insufficient or no working shown, but not from incorrect working. If both solutions given correctly in degrees (60°,240°) then can get M1M1SCB1 (max [3/4])
				[4]		
5	(a)	(ii)	$f''(x) = -\cos x - \sqrt{3}\sin x$ or…	M1	3.1a	Attempt to differentiate their f' (x) (allow sign errors but both trig functions must be changed), setting their $f''(x) = 0$ and attempting to manipulate. May see $f''(x) = -2\sin\left(x + \frac{\pi}{6}\right)$
			$\tan x = -\frac{1}{\sqrt{3}}$	A1	1.1	For correctly reaching an equation in a single trig function (oe, may see $4\sin^2 x = 1$ or $4\cos^2 x = 3$) Not from incorrect working e.g. $a - b = 0 \Rightarrow a^2 - b^2 = 0$
			$x = \frac{5}{6}\pi$ or $x = \frac{11}{6}\pi$	A1	1.1	www, for both correct in radians (allow decimals 2.62, 5.76 3sf) SC**B1** for both correct solutions (max [1/3]) if insufficient or no working shown, but not from incorrect working. Answers in degrees can get M1A1A0 (max [2/3])
				[3]		

5	**(b)**		A: $(\frac{1}{3}\pi, 0)$ AND C: $(\frac{4}{3}\pi, 0)$ B: $(\frac{5}{6}\pi, -2)$ AND D: $(\frac{11}{6}\pi, 2)$	**B1FT** **B1FT** **[2]**	**1.1** **1.1**	FT their x-values in radians or degrees from **(a)(i)**, both with y=0 Both must be correctly labelled with A and C, allow decimals 3sf FT their x-values in radians or degrees from **(a)(ii),** with both y=-2 and y=2 correct Both must be correctly labelled with B and D, allow decimals 3sf If neither mark awarded then can get one of (max [1/2]): SC**B1FT** for any two correct pairs x and y, correctly labelled SC**B1FT** for all four x-coords, correctly labelled and in ascending order (only if 2 distinct answers given in both **(a)(i)** and **(a)(ii)**) SC**B1FT** for all four y-coords, correctly labelled
5	**(c)**	**(i)**	Where the graph (of $f'(x)$) is above the x-axis or the graph is positive	**B1** **[1]**	**2.4**	Must reference the graph – do not accept just 'gradient is positive' or just 'f'(x)>0'
5	**(c)**	**(ii)**	$0 \leq x < \frac{1}{3}\pi$, $\frac{4}{3}\pi < x \leq 2\pi$ $\{x: 0 \leq x < \frac{1}{3}\pi\} \cup \{x: \frac{4}{3}\pi < x \leq 2\pi\}$ Or $[0, \frac{1}{3}\pi) \cup (\frac{4}{3}\pi, 2\pi]$	**B1** **B1** **[2]**	**2.2a** **2.5**	Both intervals correctly identified, ignore set notation for this mark cao (values must be correct) but condone any clear indication of both correct intervals e.g. $0 \rightarrow \frac{\pi}{3}$ and $\frac{4\pi}{3} \rightarrow 2\pi$ (allow decimals) Accept any combination of $\leq$ and < or ( ) and [ ] Writing their answer in correct set notation, values may be incorrect for this mark but there must be two separate intervals. Allow single-tailed inequalities as long as written in correct set notation, e.g. $\left\{x: x < \frac{\pi}{3}\right\} \cup \left\{x: x > \frac{4\pi}{3}\right\}$ Accept any combination of $\leq$ and < or ( ) and [ ] but do not accept $\cap$ instead of $\cup$.

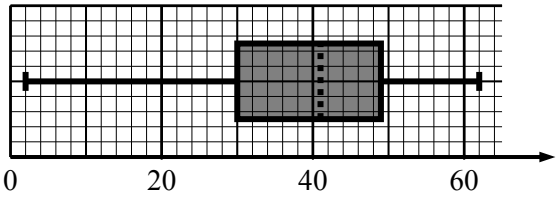
Question			Answer	Mark	AO	Guidance
6	(a)		**DR**			In this question, if candidates attempt more than one method, review all their working to identify which is their 'final' or 'substantially most complete' answer, then apply one scheme only. Candidates cannot gain marks from more than one scheme.
			$2x^2 - 6x + a = 0$	**M1**	**3.1a**	Substitute $y = x$ into equation of circle (need not be simplified – may see this quadratic in y)
			At A: $x = \frac{6 + \sqrt{36 - 8a}}{4} = \frac{3 - \sqrt{9 - 2a}}{2}$ At B: $x = \frac{3 + \sqrt{9 - 2a}}{2}$	**A1**	**2.2a**	A1 for <u>either</u> correct (condone if not specifically identified as A or B – may see $y=$ these values)
			At M: $x = \frac{3}{2}$	**A1**	**1.1**	
			$CM^2 = \left(3 - \frac{3}{2}\right)^2 + \left(\frac{3}{2}\right)^2$ $\left(= \frac{9}{2}\right)$	**M1**	**2.1**	Attempt CM using their values (method must be correct) May see $CM = \frac{3\sqrt{2}}{2}$
			$BA^2 = \left(\sqrt{9 - 2a}\right)^2 + \left(\sqrt{9 - 2a}\right)^2$ $\left(= 2\left(\sqrt{9 - 2a}\right)^2\right)$	**M1**	**1.1**	Attempt BA using their values (method must be correct) May see just $\sqrt{2(9 - 2a)}$ oe, e.g. $\sqrt{18 - 4a}$ Alternative: $AM = \sqrt{\frac{9}{2} - a}$ (and then $Area = 2 \times \frac{1}{2} \times CM \times AM$)
			Area $= \frac{1}{2} \times CM \times BA$	**M1**	**1.1**	Attempt at area, in terms of a
			$\frac{1}{2} \times \sqrt{2}\sqrt{9 - 2a} \times \frac{3}{\sqrt{2}}$ $\left(= 3\sqrt{\frac{9 - 2a}{4}}\right)$ **AG**	**A1**	**1.1**	Must see correct expression before answer

			Alternative Method			
			DR $(x-3)^2+y^2=9-a$	**M1**		Attempt complete the square for x
			C is (3, 0); radius = $\sqrt{9-a}$	**A1**		Both soi
			$CM = (3\cos 45 =)\ \frac{3}{\sqrt{2}}$	**B1**		Possibly coming from $\frac{\lvert 3-0\rvert}{\sqrt{1^2+1^2}}$
			$AM^2 = (9-a) - \left(\frac{3}{\sqrt{2}}\right)^2 \quad (= \frac{9}{2}-a)$	**M1**		Their radius^2 (in terms of a) – their CM^2
			Area = $AM \times CM$ or $\frac{1}{2}AB \times CM$	**M1**		Attempted in terms of a
			$= \sqrt{\frac{9}{2}-a} \times \frac{3}{\sqrt{2}}$ oe	**A1FT**		FT their AM or AB (in terms of a), and their CM
			$= \sqrt{\frac{9-2a}{2}} \times \frac{3}{\sqrt{2}} \quad (= 3\sqrt{\frac{9-2a}{4}}$ **AG***)*	**A1**		Must see one correct intermediate step
			Alternative method for last three marks			
			Area = $\frac{1}{2}AB \times r \times \sin BAC$	**M1**		Attempted in terms of a
			$= \sqrt{\frac{9}{2}-a} \times \sqrt{9-a} \times (\frac{3}{\sqrt{2}} \div \sqrt{9-a})$	**A1FT**		FT their r and AM or AB (in terms of a), and their CM
			$= \sqrt{\frac{9-2a}{2}} \times \frac{3}{\sqrt{2}} \quad (= 3\sqrt{\frac{9-2a}{4}}$ **AG***)*	**A1**		Must see one correct intermediate step
			Alternative method for last three marks			
			Area $= \frac{1}{2} r \times r \times \sin BCA$	**M1**		Attempted in terms of a
			$= \sqrt{9-a} \times \sqrt{9-a} \times 2\left(\sqrt{\frac{9}{2}-a} \div \sqrt{9-a}\right)\left(\frac{3}{\sqrt{2}} \div \sqrt{9-a}\right)$	**A1FT**		FT their r and AM or AB (in terms of a), and their CM (using double angle formula for sinBCA)
			$= \sqrt{\frac{9-2a}{2}} \times \frac{3}{\sqrt{2}} \quad (= 3\sqrt{\frac{9-2a}{4}})$ **AG**	**A1**		Must see one correct intermediate step
				[7]		

6	(b)	(i)	$(3\sqrt{\frac{9-2a}{4}} = 0 \Rightarrow)\ a = \frac{9}{2}$	B1 [1]	2.2a	oe
6	(b)	(ii)	$y = x$ is a tangent to the circle or A and B are coincident oe	B1 [1]	3.2a	Must see this geometrical answer (accept e.g. the line touches the circle) Accept a diagram that clearly shows the line is a tangent
6	(c)		Line $y = x$ does not meet circle	B1 [1]	3.2a	Must see this geometrical answer (condone BOD for 'the line does not touch the circle') Accept a diagram that clearly shows the line does not meet the circle

Question			Answer	Mark	AO	Guidance
7	(a)		$a^2 = 4b + 2$	M1	2.1	Setting up so that the deduction a^2 is even can be made.
			Hence a^2 is even. Hence a is even	A1	2.2a	www, must see both statements and a convincing, correct, argument oe (e.g. $a^2 = 2(2b + 1)$)
			Alternative method			
			Assume that a is odd, then a^2 is odd	M1		For setting up and stating that a is odd $\Rightarrow$ a^2 is odd May see (not required) $a = 2n+1$, $a^2 = 2(2n^2+2n)+1$ Hence a^2 is odd
			$4b$ is even, so $a^2 - 4b$ is odd Hence 2 is odd (so contradiction) Hence a is even.	A1		www, Must see both statements and a convincing, correct, argument
				[2]		
7	(b)		Assume that $a^2 - 4b = 2$ Let $a = 2n$, (where n is an integer)	M1	2.1	Setting up (must see assumption and use of a is even)
			Either of: $4n^2 - 4b = 2$ $2n^2 - 2b = 1$, Hence 1 is even (Contradiction) Hence $a^2 - 4b \neq 2$ or: $4n^2 - 4b = 2$ $n^2 - b = 0.5$, $n^2 - b$ is an integer	A1	2.1	Substituting in $a = 2n$ and correctly reaching an equation which shows a contradiction. Accept the equivalent in words if clear and correct. Also accept: $4n^2 - 4b$ is a multiple of 4, Hence $a^2 - 4b$ is a multiple of 4, which is a contradiction
			(Contradiction) Hence $a^2 - 4b \neq 2$	A1	2.2a	www, Must see both statements and a convincing, correct, argument

			Alternative method (1) Assume that $a^2 - 4b = 2$ $\Rightarrow a^2 = 4b + 2 = 2(2b + 1)$ For a to be an integer, $2b + 1$ must be even But ($2b$ is even, so) $2b + 1$ is odd (Which is a contradiction) hence $a^2 - 4b \neq 2$	 **M1** **A1** **A1**		 Must see both statements and a convincing, correct, argument www
			Alternative method (2) a even $\Rightarrow a^2 = 4n$ (n an integer) $\Rightarrow a^2$ is congruent to 0 mod 4 $4b + 2$ is congruent to 2 mod 4 Therefore a^2 cannot equal $4b + 2$	 **M1** **A1** **A1**		 Must see previous two lines and a convincing, correct, argument www
			Alternative method (3) Assume that $a^2 - 4b = 2$, then a is even (and consider whether b is odd or even) If b is odd then 2 is either 0 or a multiple of 4, (so contradiction) AND If b is even then 2 is either 0 or a multiple of 4, (so contradiction) Therefore a^2 cannot equal $4b + 2$	 **M1** **A1** **A1**		 For setting up using part **(a)** and considering either case where b is odd or even (ignore any reference to the cases where a is odd as these are not required) May see (but not required) $a=2n$ so $a^2=4n^2$ Condone using the same letter (e.g. n) in a and b for this mark only. For correctly considering both cases either algebraically or in words. May see (but not required) $b = 2m+1$, so $a^2 - 4b = 4(n^2 - (2m+1))$ And $b = 2m$, so $a^2 - 4b = 4(n^2 - 2m)$ Do not award this mark if same integer (e.g. n) used in both a and b A fully correct, convincing argument with conclusion, www.
				[3]		

Question			Answer	Mark	AO	Guidance
8	(a)		Median = 41	B1	1.1	Stated or shown on diagram (need not be labelled)
			Quartiles = 30, 49	B1	1.1	Stated or shown on diagram (need not be labelled)
				B1FT	1.2	Correct diagram, FT **their** median & quartiles & **correct** end points shown (2,62) – all correct to +/-0.5 (Must see a box with a vertical line and two whiskers)
				B1	1.2	An appropriate linear scale, consistently labelled. (This mark can only be awarded if a box-and-whisker plot is drawn)
				[4]		
8	(b)		Using $Q_1 - 1.5 \times IQR$ or $Q_3 + 1.5 \times IQR$	M1	1.1	Attempting to calculate either with their quartiles (may be implied by 1.5 or 77.5)
			$30 - 1.5 \times (49 - 30) = 1.5$ AND $49 + 1.5 \times (49 - 30) = 77.5$	A1	1.1	Must see both correct calculations or both 1.5 and 77.5
			So no (lower) outliers and no (upper) outliers	A1	2.3	www (so must be from correct calculations) Accept a single statement of 'no outliers' following correct calculations oe e.g. 'no heights should be ignored'
			Alternative using mean and sd:			
			$\mu \pm 2\sigma = 37.7 \pm 2 \times 16.0$	M1		Calculating either $\mu \pm 2\sigma$ with sensible values $\mu \in [35,40]$ $\sigma \in [14,19]$
			$\mu + 2\sigma = 69.6$ (3sf) Hence no (upper) outliers	A1		Must see correct calculation or value and statement
			$\mu - 2\sigma = 5.83$ (3sf) Hence one (lower) outlier (2)	A1		Must see correct calculation or value and statement
				[3]		

Question			Answer	Mark	AO	Guidance
9	(a)		B$(20, \frac{1}{3})$	B1 [1]	3.3	Allow '$n = 20, p = \frac{1}{3}$' or just $(20, \frac{1}{3})$. Accept $p = 0.\dot{3}$ but not 0.333
9	(b)	(i)	${}^{20}C_x(\frac{2}{3})^{(20-x)}(\frac{1}{3})^x$	B1FT [1]	3.4	FT their values of n and p and allow $\binom{20}{x}$ as alternative to ${}^{20}C_x$ (If decimals used both must be correct to 3sf)
9	(b)	(ii)	$\{x : x \in \mathbb{Z}, 0 \leq x \leq 20\}$	B1FT [1]	1.2	FT their '20' only Any acceptable alternative in set notation Indication that x is an integer must be present (but condone 'integer' or 'whole number' or $x \in \mathbb{N}$)
9	(c)		P$(X \leq 9)$ − P$(X \leq 4)$ attempted (= 0.90810 − 0.15151) = 0.757 (3 sf)	M1 A1 [2]	3.4 1.1	Condone P$(X \leq 9)$ − P$(X \leq 5)$, or 0.90810 − 0.29721 or 0.611 May be implied by correct working or answer. Allow sum of P$(X = x)$ for x from 5 to 9. **BC** cao
9	(d)		May include too many from one year and too few from another year	B1 [1]	3.5b	For an explanation (which must be in context) showing clear recognition that a random sample (of this size) would not be representative across the year groups. Acceptable similar answers in context include: • 'should include roughly equal numbers from each year' • 'not representative of each year group' • 'may not include students from every year group' Do not accept 'may end up with more students from Y7-11 than from Y12-13' (because a representative sample would have this) Must be in context – so do not accept generic answers without context such as just: 'It might be biased' or 'It's not representative' or 'it should be a stratified sample' Do not accept answers solely referring to the suitability of the individual students within the sample to represent the school

Question			Answer	Mark	AO	Guidance
10			Assumption: sd for this river is 0.25	B1	3.3	For a correct assumption, in context, that is necessary and specific to 'the population' or 'the river'. Acceptable answers include: • sd for **this river** is the same as for the UK • assume the population of **this river** is normally distributed • mass of fish in **this river** is normally distributed Do not accept answers that are: • referencing the sample (e.g. bias) – question states random • not about this river e.g. 'in the UK' • generic e.g. 'masses of fish are normally distributed' Candidates must show recognition that the assumption is needed about the (sub-)population of this river. The mean and sd for the whole of the UK are given in the question. Condone the sample mean of fish from this river $\bar{X} \sim N$ but references to X or $\bar{X}$ alone are not enough unless defined in context. Ignore all else (e.g. ignore any statements about the mean)
			Allow 2 sf throughout			Subtract B1 for each error eg: 2-tail B1B0 undefined μ B1B0 not in terms of parameter B1B0 μ = sample mean implied B1B0 Not include value 4.2 B0B0 eg $H_0 = 4.2$ etc: B0B0 Allow any letter for μ (except $X, \bar{X}$: B0B0) Condone 'average' for 'mean' but do not accept definitions for μ that are clearly not about this river e.g. 'the UK' B1B0
			H_0: $\mu = 4.2$; H_1: $\mu < 4.2$ where μ = (population) mean (mass) (of this river)	B1B1	1.1 2.5	
			$N(4.2, \frac{0.25^2}{100})$ & $\bar{X} < 4.16$	M1*	3.3	This mark may be implied by the correct value of 0.0548 or 0.945 (correct to 2sf) (even if within incorrect statement eg $P(X = 4.16) = 0.0548$) Condone >, =, ≥, ≤
			$P(\bar{X} < 4.16) = 0.0548$	A1	3.4	**BC** (awrt 0.055 2sf)

			0.0548 > 0.05	**A1FT**	**1.1**	FT correct comparison **for their value** as long as consistent with their test (e.g. 2-tail 0.025 or 0.975) Must be seen, allow on diag A0 if the comparison is not for their value (e.g. if miscopied)
			$\frac{a-4.2}{0.25/10} = 1.645$ $a = 4.159$ or CV is 4.159 $4.16 > 4.159$	**M1*** **A1*** **A1 dep***		 cao dep A1* Must be seen
			$\frac{4.16-4.2}{0.25/10}$ $= -1.6$ $-1.6 > -1.645$ or $1.6 < 1.645$	**M1*** **A1*** **A1 dep***		 cao dep A1* Must be seen
			Do not reject H_0 Allow Accept H_0 Insufficient evidence (at 5%) that (mean) mass is less than in UK	**M1 dep*** **A1**	**1.1** **2.2b**	dep M1* Correct conclusion about H_0 for their comparison, provided they have compared with an appropriate value. Accept Reject H_1 or Insufficient (or No) evidence to reject H_0 www (all preceding calculations must be correct, i.e. dependent on all previous M and A marks) Must be in context and not definite. Acceptable answers include: • 'no evidence that (mean) mass is less' • 'insufficient evidence to say that the environmentalist is correct' • 'insufficient evidence that the fish in this river are smaller' Do not accept: • 'therefore the mean is 4.2' (definite) • 'the mean has decreased' (incorrect reference to change over time) • 'the mean is not less' or 'the mean is the same' (insufficient evidence does not mean that the statement for H_0 is true)
				[8]		

Question			Answer	Mark	AO	Guidance
11	(a)		0.841	B1 [1]	1.2	Allow $\frac{5}{6}$ or 0.84 (2 sf) (from "rule-of-thumb") awrt 0.84
11	(b)		$\frac{45-\mu}{\sigma} = \phi^{-1}(0.8)$ $\quad$ $\frac{25-\mu}{\sigma} = -\phi^{-1}(0.7)$	M1	3.1a	One of these attempted. Or P($Z < \frac{45-\mu}{\sigma}$) = 0.8 seen or this standardised form clearly shown on a diagram
			$\frac{45-\mu}{\sigma} = 0.84162$	A1	1.1	Not a required answer so allow truncated e.g. 0.841, 0.84…
			$\frac{25-\mu}{\sigma} = -0.52440$	A1	1.1	Not a required answer so allow truncated e.g. -0.52…
			$\frac{45-\mu}{25-\mu} = -\frac{0.84162}{0.52440}$ $\quad$ (= –1.60492)	M1 dep	2.1	dep previous M1. Attempt to solve their equations simultaneously, any method (may be implied by correct answers)
			$\mu \in [32.6, 32.8]$ **and** $\sigma \in [14.5, 14.7]$	A1	1.1	For obtaining both values in the given intervals.
			$\mu = 32.7$ (3 sf) **and** $\sigma = 14.6$ (3 sf)	A1 [6]	1.1	For both μ and σ correct to 3sf **BC** cao ($\mu = 32.6778$, $\sigma = 14.6411$) SC**B1** for either $\mu = 32.7$ (3sf) or $\sigma = 14.6$ (3sf) if neither A mark gained, but must be correct to 3sf. (max [5/6])
11	(c)		$\frac{b-10}{2} = -\frac{c-12}{3}$	M1	3.1a	oe but signs must be correct (e.g. accept $\frac{b-10}{2} = \frac{12-c}{3}$)
			$b = 18 - \frac{2}{3}c$	A1 [2]	1.1	oe but must have b as the subject

Question			Answer	Mark	AO	Guidance
12	(a)			M1	3.3	For indication of B(100, 1/6) used and an attempt at any P($X \leq a$) in the right region, i.e. finding P($X \leq a$) for any $a \leq 16$ **BC**
			P($X \leq 11$) = 0.0777 (3 sf) P($X \leq 10$) = 0.0427 (3 sf)	M1	3.4	For finding both P($X \leq 10$) and P($X \leq 11$) **BC** (allow numerical slip if clear indication that the right distribution is used) Allow < in both cases if values are correct.
			Largest value of X is 10	A1	3.4	Dep on both M marks so must have found both P($X \leq 11$) and P($X \leq 10$) correctly (condone values correct to 2sf throughout but do not accept answers that are not correct to 2sf, e.g. 0.077 or 0.042 unless there is a clear indication of truncation such as '…')
				[3]		
12	(b)		0.0427 (3 sf)	B1FT	3.4	FT their P($X \leq 10$) (allow 2sf) Accept 4.27%
				[1]		

Question			Answer	Mark	AO	Guidance
13	(a)	(i)	$k = 70$ to 80 inclusive	B1 [1]	1.2	
13	(a)	(ii)	100 − their k	B1FT [1]	2.2a	Strictly FT their k (i.e. this must be 100 – their k only)
13	(b)	(i)	The group with highest usage of private (motorised) transport (top left), because private transport uses more fuel than public transport (for the same travel distance, per person).	B1 [1]	2.2b	For a clear explanation that must both identify the group unambiguously (e.g. 'top left', 'least public transport use') and give a reason. Accept equivalent justifications e.g. 'each individual uses more fuel'
13	(b)	(ii)	e.g. Lengths of journeys (or distance travelled) and e.g. how many people travel in each vehicle used (or 'occupancy' of each mode)	B1 B1 [2]	2.4 2.4	For two sensible distinct suggestions. Acceptable answers include: • The type or amount of fuel used (by different modes of transport) • Proportion or usage of Electric Vehicles • Types of vehicle used (or available) • Proportion of the different modes of transport within each category • The occupancy of each mode (or e.g. car sharing) • Proportion of full-time vs part-time working patterns Do not accept: • Population size (because F is per person) • Number of those not in work (because F is for employees) • References to emissions e.g. 'given off' (because F is the amount of fuel used)

13	(c)	(i)	Large percentage walk/cycle/work from home Small area	B1 B1 [2]	2.2b 2.4	For the shared characteristic (any of walk/cycle/work from home). Condone 'these have the lowest total proportion using public or motorised private transport combined' but do not accept only 'lower percentage using motorised private transport' or 'lower percentage using public transport' – need both For a justification (must be related to the LA). Acceptable answers include: • 'less need to travel to work' • 'shorter journeys' • 'better walking/cycling infrasutructure' But not just 'more work from home' (this is the characteristic) (*A* is Scilly Isles, *B* is City of London)
13	(c)	(ii)	*A* is rural, *B* is urban Public transport is absent in *A* but used in *B* or, eg, Those in *B* who don't walk, don't use cars, so there is probably a lot of traffic, so *B* is urban. No public transport in *A* so *A* is rural	B1 B1 [2]	2.2b 2.4	B1 for identifying the difference in environment (must make a comparison e.g. '*A* is more rural than *B*'). Accept clearly equivalent statements e.g. 'city' or 'countryside' B1 for justification. Cannot just restate the data so do not accept e.g. '*A* has low public transport use' – must give a justification as to why this might be. Accept e.g. 'less availability of public transport' Other sensible answers may be seen e.g. 'motorised private transport is almost absent in *B* but more widely used in *A*'
13	(d)		Some points are too close together to read	B1 [1]	2.3	Must make a specific criticism of the graph related to reading values. Acceptable answers include: • 'the scale is not precise enough to read detailed values' • 'closely clustered points mean it is hard to read' • 'there are no gridlines so cannot read exact values' Do not accept generic statements such as 'the data may not be accurate' or references to information not included on the graph.

Question			Answer	Mark	AO	Guidance
14	(a)		P(has disease | positive result) $= \frac{\text{P(has disease \& positive result)}}{\text{P(positive result)}}$	M1	3.4	Attempting this calculation, allow wrong values but for this mark must be a fraction with a product in the numerator and a sum of two products in the denominator.
			$= \frac{0.35\times0.95}{0.35\times0.95+0.65\times0.1}$	A1	1.1	Fully correct expression
			= 0.836 (3 sf)	A1 [3]	1.1	Or 133/159 or 0.8365 (4sf) (0.836477…)
14	(b)		(Let proportion having the disease = p) $p \times 0.95 + (1 - p) \times 0.1$	M1	1.1	Setting up an expression in this form using the given values
			$p \times 0.95 + (1 - p) \times 0.1 = 0.43$ $0.85p = 0.33$	M1	3.4	Setting their expression =0.43 and attempting to solve
			$p = 0.388$	A1	1.1	cao (watch for 0.389 from incorrect working)
			About 39% of trees (in county *B*) have the disease	B1FT [4]	3.2a	"Around 38.8 or 39 or 40" (oe e.g. 2/5). Must be in context and include "about" or "approximately" or "roughly" oe

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